高瞻計畫_振動學課程
Lecture 1: Single Degree of Freedom Systems (III)

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Outline

- Rotordynamics, an introduction
- Impulse Responses
- Arbitrary excitation
- Transfer function and Laplace Domain
- Shock Isolation
- Simple problems
- Youtube Demos

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Part I: Rotordynamics: an Introduction

- Rotor phenomenon
- Unbalance
- Campbell diagrams

Rotordynamics

- A specialized branch concerned with the behavior and diagnosis of rotating structures.
  - commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage.
  - Vibration, noise, bearing damages
- Key issues to be introduced
  - Critical speed
  - Whirling
  - Campbell diagram

Jeffcott Rotors

- A single disk mounted on a flexible and massless shaft with rigid bearings
- Serves as the fundamental model for studying rotor phenomena

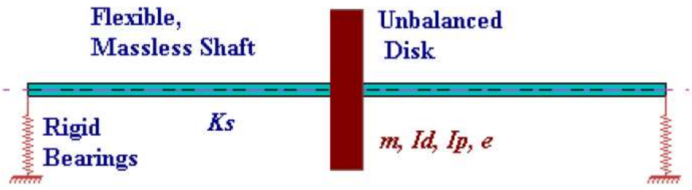

Figure 3.1-1 A simple Laval-Jeffcott rotor

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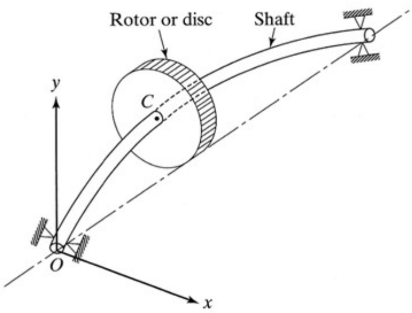
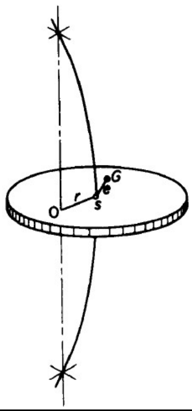
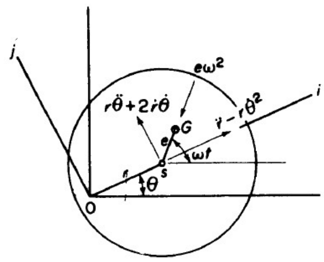
Whirling

- Rotation of bent shaft
- ω: rotating speed, $\dot{\theta}$: whirling speed







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Critical Speeds

- When frequency of rotation of shaft = one of the natural frequencies of the shaft, critical speed of undamped system:

$$\omega_n = \sqrt{\frac{k}{m}}$$

- When $\omega = \omega_n$, rotor undergoes large deflections
  $\rightarrow$ cause fatique and damage bearings
- Slow transition of rotating shaft through the critical speed aids development of large amplitudes.

- Whirling critical speed should not be below 115 percent of the design full power speed (NOAA)

www.omao.noaa.gov/swath/contractdocs/attachments/AttachmentJ-02Rev2.pdf
Marine & Aviation Operaitons, National Oceanic & Atmospheric Administration, US Dept of Commerce

Whirling: Mathematics

$$\mathbf{a}_G = [(\ddot{r} - r\dot{\theta}^2) - e\omega^2\cos(\omega t - \theta)]\mathbf{i} + [(r\ddot{\theta} + 2\dot{r}\dot{\theta}) - e\omega^2\sin(\omega t - \theta)]\mathbf{j}$$

$$-kr - c\dot{r} = m[\ddot{r} - r\dot{\theta}^2 - e\omega^2\cos(\omega t - \theta)]$$

$$-cr\dot{\theta} = m[r\ddot{\theta} + 2\dot{r}\dot{\theta} - e\omega^2\sin(\omega t - \theta)]$$

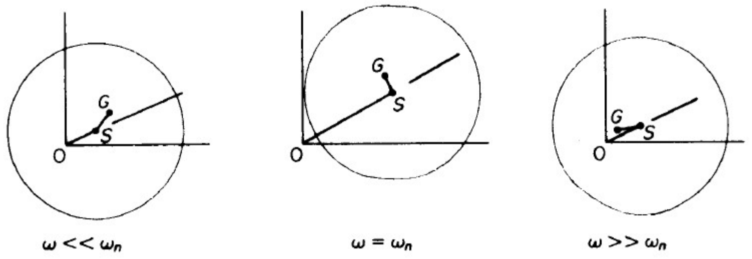
$\Rightarrow$

$$\ddot{r} + \frac{c}{m}\dot{r} + \left(\frac{k}{m} - \dot{\theta}^2\right)r = e\omega^2\cos(\omega t - \theta)$$

$$r\ddot{\theta} + \left(\frac{c}{m}r + 2\dot{r}\right)\dot{\theta} = e\omega^2\sin(\omega t - \theta)$$

Whirling Behavior

- Synchronous whirling: $\dot{\theta} = \omega$



As rotating speed passes the critical speed, the imbalance location would actually move toward the center

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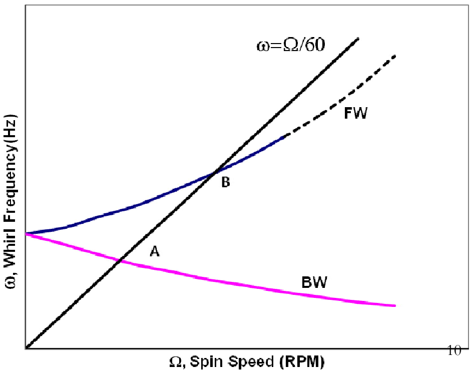
Campbell Diagram

- known as "Whirl Speed Map" or a "Frequency Interference Diagram“
- Basic concept
  - Natural frequencies depends on rotating speed
  - Resonance occurs as natural frequencies hit rotating speed



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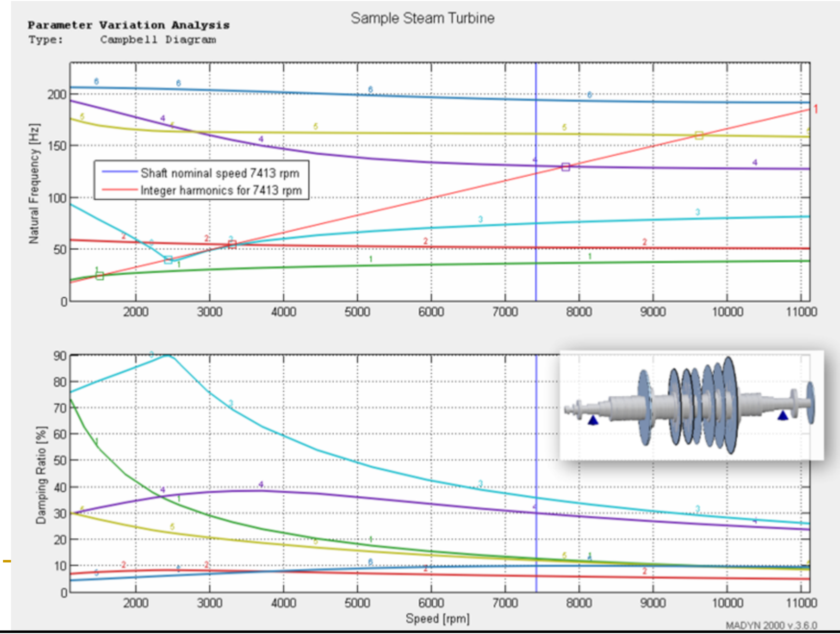
Turbine Example



Dynamic Balancing

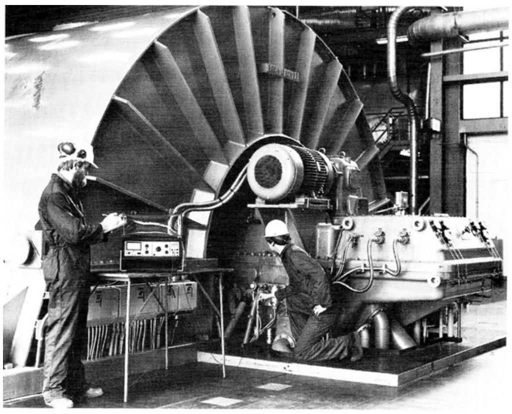


FIGURE 9.10 Two-plane balancing. (Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough. Mass.)

Part II: Periodic Inputs

Introduction

- In Lecture III, we have introduced the response of a SDOF system subjected to a single sinusoidal response
- How about the responses subjected to
  - A general periodic input
    - E.g., a saw tooth or a rectangular pulse train
  - A non-periodic input

Responses Under a General Force (I)

$$F(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos j\omega t + \sum_{j=1}^{\infty} b_j \sin j\omega t$$

$$a_j = \frac{2}{\tau}\int_0^{\tau} F(t)\cos j\omega t\,dt, \qquad j = 0, 1, 2, \ldots$$

$$b_j = \frac{2}{\tau}\int_0^{\tau} F(t)\sin j\omega t\,dt, \qquad j = 1, 2, \ldots$$

$$m\ddot{x} + c\dot{x} + kx = F(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos j\omega t + \sum_{j=1}^{\infty} b_j \sin j\omega t$$

$$m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2}$$

$$m\ddot{x} + c\dot{x} + kx = a_j \cos j\omega t$$

$$m\ddot{x} + c\dot{x} + kx = b_j \sin j\omega t$$

$$x_p(t) = \frac{a_0}{2k}$$

Responses Under a General Force (II)

$$\phi_j = \tan^{-1}\left(\frac{2\zeta jr}{1 - j^2 r^2}\right)$$

$$r = \frac{\omega}{\omega_n}$$

$$x_p(t) = \frac{a_0}{2k} + \sum_{j=1}^{\infty} \frac{(a_j/k)}{\sqrt{(1 - j^2r^2)^2 + (2\zeta jr)^2}}\cos(j\omega t - \phi_j) + \sum_{j=1}^{\infty} \frac{(b_j/k)}{\sqrt{(1 - j^2r^2)^2 + (2\zeta jr)^2}}\sin(j\omega t - \phi_j)$$

Response Under a Periodic Force with Irregular Form

$$a_0 = \frac{2}{N}\sum_{i=1}^{N} F_i$$

$$a_j = \frac{2}{N}\sum_{i=1}^{N} F_i \cos\frac{2j\pi t_i}{\tau}, \qquad j = 1, 2, \ldots$$

$$b_j = \frac{2}{N}\sum_{i=1}^{N} F_i \sin\frac{2j\pi t_i}{\tau}, \qquad j = 1, 2, \ldots$$

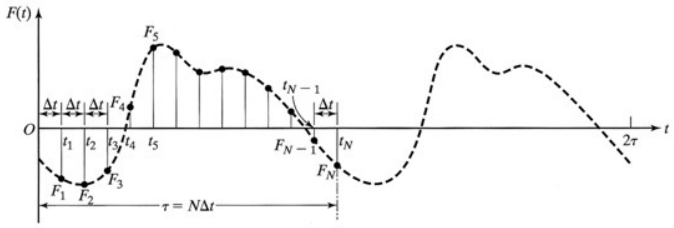


FIGURE 4.2 An irregular forcing function.

Part III: Vibration Subject to Arbitrary Excitations

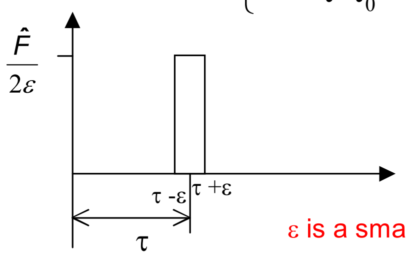
Introduction

- It is important to evaluate the response to a general, non-periodic input
- No, exact analytical solutions available
- However, the task can be performed by either
  - Convolution approach
  - Fourier transform or Laplace transform approach
    - Transfer function

Impulse Response

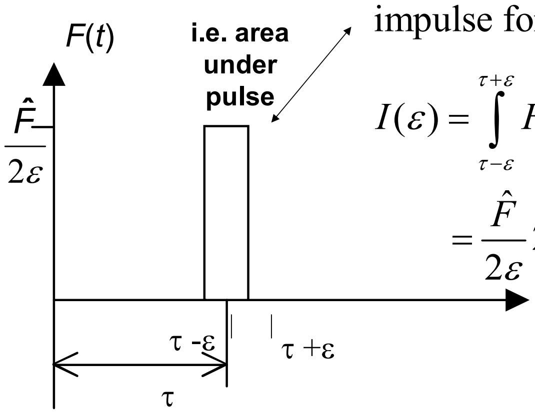
- The response of a vibration system subjected to a unit impulse input
- Impulse function

$$\delta(t-t_0)=\begin{cases}0 & t \neq t_0 \\ \infty & t=t_0\end{cases} \qquad \int_{-\infty}^{\infty}\delta(t-t_0)dt=1$$



ε is a small positive number

From sophomore dynamics:



$$\text{impulse force} = \int F(t)dt = F\Delta t$$

$$I(\varepsilon) = \int_{\tau-\varepsilon}^{\tau+\varepsilon} F(t)dt = \int_{-\infty}^{\infty} F(t)dt \ \mathrm{N \cdot s}$$

$$= \frac{\hat{F}}{2\varepsilon} 2\varepsilon = \hat{F}$$

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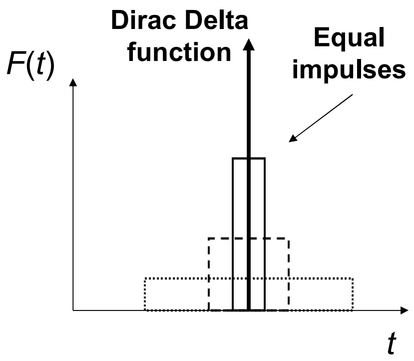
Use these properties to define the impulse function:



$$F(t-\tau) = 0, \quad t \neq \tau$$

$$\int_{-\infty}^{\infty} F(t-\tau)dt = \hat{F}$$

If $\hat{F} = 1$, this is the Dirac Delta $\delta(\mathrm{t})$

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Effect on spring-mass-damper ?

Just after impulse — Just before impulse

$$\overbrace{F\Delta t = \Delta mv}^{\text{impulse= momentum change}} \quad = m[v(t_0^+) - v(t_0^-)]$$

$$\hat{F} = mv_0 \Rightarrow v_0 = \frac{\hat{F}}{m} = \frac{F\Delta t}{m}$$

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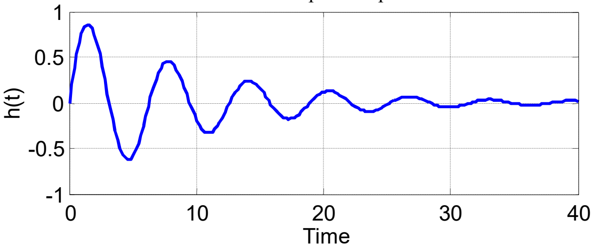
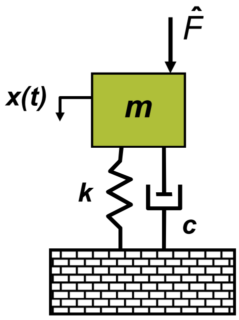
For an underdamped system

$$x(t) = \frac{\hat{F}e^{-\zeta\omega_n t}}{m\omega_d}\sin\omega_d t \quad \text{(response with zero I.C.)}$$

$$x(t) = \hat{F}h(t), \text{ where } \underbrace{h(t) = \frac{e^{-\zeta\omega_n t}}{m\omega_d}\sin\omega_d t}_{\text{impulse response function}}$$





$$h(t-\tau) = \frac{1}{m\omega_n}\sin\omega_n(t-\tau)$$

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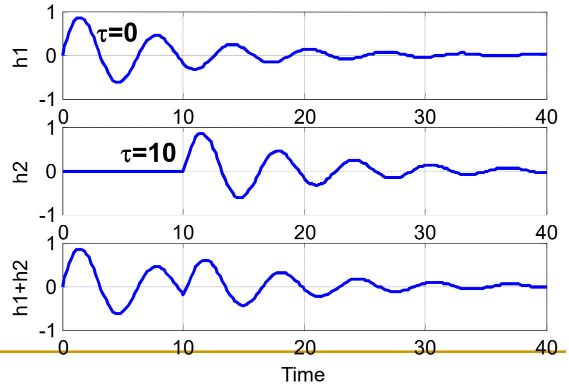
$$h(t-\tau)=\begin{cases}0 & t<\tau\\ \dfrac{e^{-\zeta\omega_n(t-\tau)}}{m\omega_d}\sin\omega_d(t-\tau) & t>\tau\end{cases}$$

for the case that the impulse occurs at τ

note that the effects of non-zero initial conditions and other forcing terms must be super imposed on this solution

For example: If two pulses occur at two different times then their impulse responses will superimpose



Impulse Response of a Vibration System (I)

$$\text{Impulse} = F\Delta t = m\dot{x}_2 - m\dot{x}_1$$

$$\underset{\sim}{F} = \int_t^{t+\Delta t} F\,dt$$

$$\underset{\sim}{f} = \lim_{\Delta t\to 0}\int_t^{t+\Delta t} F\,dt = F\,dt = 1$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

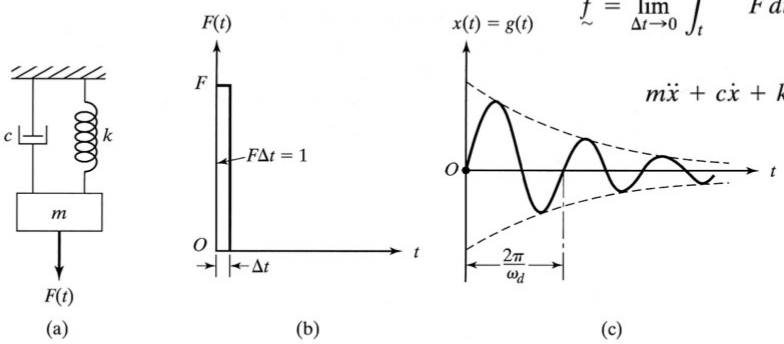


FIGURE 4.3 A single degree of freedom system subjected to an impulse.

Impulse Response of a Vibration System (II)

$$x(t) = e^{-\zeta\omega_n t}\left\{x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d} \sin \omega_d t\right\}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\text{Impulse} = \underset{\sim}{f} = 1 = m\dot{x}(t=0) - m\dot{x}(t=0^-) = m\dot{x}_0$$

$$x(t=0) = x_0 = 0$$

$$\dot{x}(t=0) = \dot{x}_0 = \frac{1}{m}$$

$$x(t) = g(t) = \frac{e^{-\zeta\omega_n t}}{m\omega_d} \sin \omega_d t$$

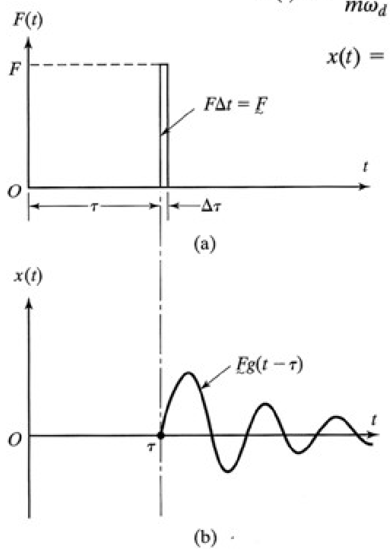
Convolution Integrals: Introduction

- Decompose a general input to the combination of a series of impulse train
- Superimpose the impulse response to form the final response
- The method itself can be treated as an “analytical” relation

Convolution Integral (I)

$$x(t) = \frac{\underset{\sim}{F}\, e^{-\zeta\omega_n t}}{m\omega_d} \sin \omega_d t = \underset{\sim}{F}\, g(t)$$

$$x(t) = \underset{\sim}{F}\, g(t - \tau)$$



Convolution Integral (II)

$$\Delta x(t) = F(\tau)\,\Delta\tau\, g(t - \tau)$$

$$x(t) \simeq \sum F(\tau) g(t - \tau)\,\Delta\tau$$

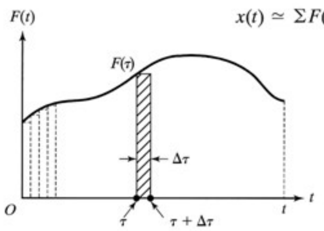


FIGURE 4.6 An arbitrary (nonperiodic) forcing function.

$$x(t) = \int_0^t F(\tau) g(t - \tau)\, d\tau$$

$$x(t) = \frac{1}{m\omega_d}\int_0^t F(\tau) e^{-\zeta\omega_n (t-\tau)} \sin \omega_d (t - \tau)\, d\tau$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

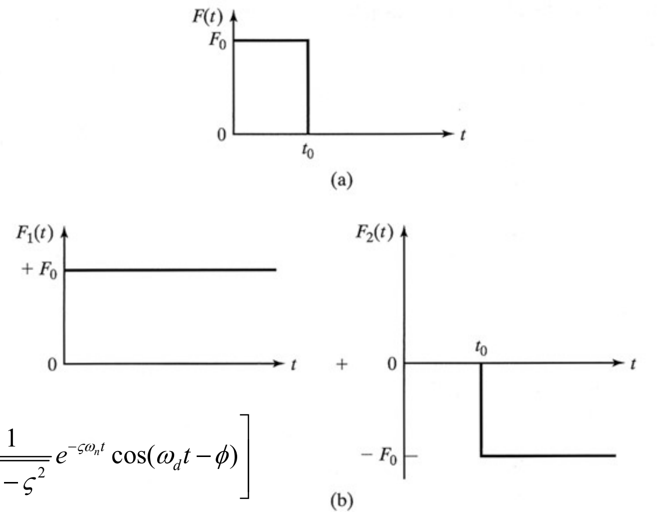
$$m\ddot{x} + c\dot{x} + kx = F$$

$$z(t) = -\frac{1}{\omega_d}\int_0^t \ddot{y}(\tau)\, e^{-\zeta\omega_n(t-\tau)} \sin \omega_d (t - \tau)\, d\tau$$

Linear Superposition Method

- A complicate input can be decomposed into a few simple input with
  - scaling, multiplexing, and time shift operations
- The response are then becomes the linear superposition of the simple output after these linear operations

Example: Pulse Input



$$x(t) = \frac{F_0}{k}\left[1 - \frac{1}{\sqrt{1-\varsigma^2}} e^{-\varsigma\omega_n t} \cos(\omega_d t - \phi)\right]$$

$$x(t) = \frac{-F_0}{k}\left[1 - \frac{1}{\sqrt{1-\varsigma^2}} e^{-\varsigma\omega_n (t-t_0)} \cos(\omega_d (t - t_0) - \phi)\right]$$

Time Shift Operation

- E.g., for the pulse input
  - The total response = x1 + x2
  - Where
  - X1

$$x(t)=\frac{F_0}{k}\left[1-\frac{1}{\sqrt{1-\varsigma^2}}e^{-\varsigma\omega_n t}\cos(\omega_d t-\phi)\right]$$

  - x2

$$x(t)=\frac{-F_0}{k}\left[1-\frac{1}{\sqrt{1-\varsigma^2}}e^{-\varsigma\omega_n (t-t_0)}\cos(\omega_d (t-t_0)-\phi)\right]$$

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Part IV: Transfer Function and Laplace Transform

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Transfer Functions

- A black box approach to correlate the input-output relation
- Based on linear superposition principle
- Usually performed by Laplace Transform approach
  - Or Fourier Transform

Laplace Transform

- **Changes ODE into algebraic equation**
- **Solve algebraic equation then compute the inverse transform**
- **Rule and table based in many cases**
- **Is used extensively in control analysis to examine the response**
- **Related to the frequency response function**

$$X(s) = \mathcal{L}(x(t)) = \int_0^\infty x(t)e^{-st}dt$$

Take the transform of the equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos\omega t \Rightarrow$$

$$(ms^2 + cs + k)X(s) = \frac{F_0 s}{s^2 + \omega^2}$$

Now solve algebraic equation in *s* for *X*(*s*)

$$X(s) = \frac{F_0 s}{(ms^2 + cs + k)(s^2 + \omega^2)}$$

To get the time response this must be "inverse transformed"

Laplace Transform: Fundamentals (I)

Consider a SDOF vibration system

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$\bar{x}(s) = \mathcal{L}x(t) = \int_0^\infty e^{-st}x(t)\,dt$$

$$\mathcal{L}\frac{dx}{dt}(t) = e^{-st}x(t)\Big|_0^\infty + s\int_0^\infty e^{-st}x(t)\,dt = s\bar{x}(s) - x(0)$$

$$\mathcal{L}\frac{d^2x}{dt^2}(t) = \int_0^\infty e^{-st}\frac{d^2x}{dt^2}(t)\,dt = s^2\bar{x}(s) - sx(0) - \dot{x}(0)$$

$$\bar{F}(s) = \mathcal{L}F(t) = \int_0^\infty e^{-st}F(t)\,dt$$

Laplace Transform: Fundamentals (II)

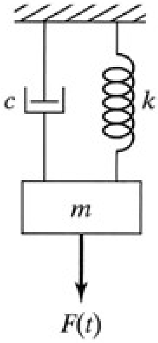
$$(ms^2 + cs + k)\,\overline{x}(s) = \overline{F}(s) + m\dot{x}(0) + (ms + c)x(0)$$

$$\overline{Z}(s) = \frac{\overline{F}(s)}{\overline{x}(s)} = ms^2 + cs + k$$

$$\overline{Y}(s) = \frac{1}{\overline{Z}(s)} = \frac{\overline{x}(s)}{\overline{F}(s)} = \frac{1}{ms^2 + cs + k} = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\overline{x}(s) = \overline{Y}(s)\,\overline{F}(s)$$

$$x(t) = \mathscr{L}^{-1}\overline{x}(s) = \mathscr{L}^{-1}\overline{Y}(s)\overline{F}(s)$$



Transfer Functions: Z(s) Impedance; Y(s): Admittance

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Typical Laplace Transforms

$F(s)$	$f(t), t \geq 0$
1. 1	$\delta(t)$, unit impulse at $t = 0$
2. $\frac{1}{s}$	$u_s(t)$, unit step
3. $\frac{n!}{s^{n+1}}$	t^n
4. $\frac{1}{s+a}$	e^{-at}
5. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$
6. $\frac{a}{s(s+a)}$	$1 - e^{-at}$
7. $\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}(e^{-at} - e^{-bt})$
8. $\frac{s+p}{(s+a)(s+b)}$	$\frac{1}{b-a}[(p-a)e^{-at} - (p-b)e^{-bt}]$
9. $\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-b)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$
10. $\frac{s+p}{(s+a)(s+b)(s+c)}$	$\frac{(p-a)e^{-at}}{(b-a)(c-a)} + \frac{(p-b)e^{-bt}}{(c-b)(a-b)} + \frac{(p-c)e^{-ct}}{(a-c)(b-c)}$
11. $\frac{b}{s^2+b^2}$	$\sin bt$
12. $\frac{s}{s^2+b^2}$	$\cos bt$
13. $\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
14. $\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
15. $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t \quad \zeta < 1$
16. $\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 + \frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}t + \phi\right) \quad \zeta < 1$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} + \pi$ (third quadrant)

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Properties of Laplace Transform

$f(t)$	$F(s) = \int_0^\infty f(t)e^{-st}\,dt$
1. $af_1(t) + bf_2(t)$	$aF_1(s) + bF_2(s)$
2. $\frac{df}{dt}$	$sF(s) - f(0)$
3. $\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0) - \left.\frac{df}{dt}\right\|_{t=0}$
4. $\frac{d^nf}{dt^n}$	$s^nF(s) - \sum_{k=1}^{n} s^{n-k}g_{k-1}$ $g_{k-1} = \left.\frac{d^{k-1}f}{dt^{k-1}}\right\|_{t=0}$
5. $\int_0^t f(t)\,dt$	$\frac{F(s)}{s} + \frac{h(0)}{s}$ $h(0) = \left.\int f(t)\,dt\right\|_{t=0}$
6. $g(t) = \begin{cases} 0 & t < D \\ f(t-D) & t \geq D \end{cases}$	$G(s) = e^{-sD}F(s)$
7. $e^{-at}f(t)$	$F(s+a)$
8. $tf(t)$	$-\frac{dF(s)}{ds}$
9. $f(t) = \int_0^t x(t-\tau)y(\tau)\,d\tau = \int_0^t y(t-\tau)x(\tau)\,d\tau$	$F(s) = X(s)Y(s)$
10. $f(\infty) = \lim_{s\to 0} sF(s)$	
11. $f(0+) = \lim_{s\to\infty} sF(s)$	

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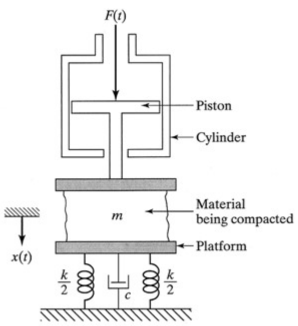
Example: Rao Ex. 4.16

Find the response



$$F(t) = \begin{cases} F_0 & \text{for } 0 \leq t \leq t_0 \\ 0 & \text{for } t > t_0 \end{cases}$$

$$\bar{F}(s) = \mathscr{L}\,F(t) = \frac{F_0(1 - e^{-t_0 s})}{s}$$

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$$\bar{x}(s) = \frac{\bar{F}(s)}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)} + \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} x_0 + \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \dot{x}_0$$

$$\bar{x}(s) = \frac{F_0(1 - e^{-t_0 s})}{ms(s^2 + 2\zeta\omega_n s + \omega_n^2)} + \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} x_0 + \frac{1}{s^2 + 2\zeta\omega_n + \omega_n^2} \dot{x}_0$$

$$= \frac{F_0}{m\omega_n^2} \frac{1}{s\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right)} - \frac{F_0}{m\omega_n^2} \frac{e^{-t_0 s}}{s\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right)} + \frac{x_0}{\omega_n^2} \frac{s}{\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right)} + \left(\frac{2\zeta x_0}{\omega_n} + \frac{\dot{x}_0}{\omega_n^2}\right) \frac{1}{\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right)}$$

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$$x(t) = \frac{F_0}{m\omega_n^2\sqrt{1 - \zeta^2}}\left[- e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \phi_1) + e^{-\zeta\omega_n (t - t_0)} \sin\{\omega_n \sqrt{1 - \zeta^2}(t - t_0) + \phi_1\}\right] - \frac{x_0}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t - \phi_1) + \frac{(2\zeta\omega_n x_0 + \dot{x}_0)}{\omega_n\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t)$$

where $\phi_1 = \cos^{-1}(\zeta)$

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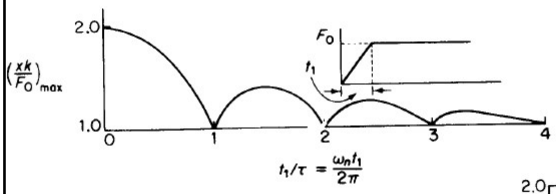
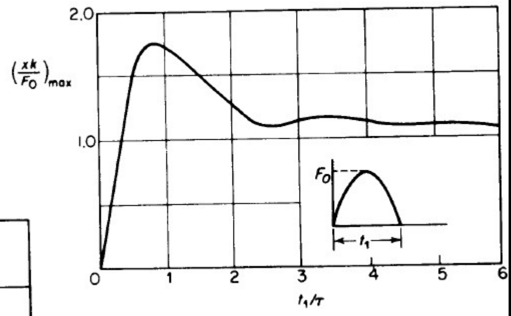
Part V: Response Spectrum

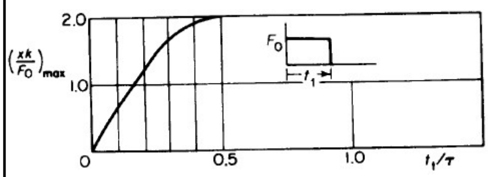
Introduction

- Shock
  - A sudden application of a force input to a SDOF system to result a transient response
  - The maximum value of the response can be used to measure the shcok sensitivity
  - Response spectrum is a plot of the maximum peak response of the SDOF oscillator as a function of natural frequency
  - Different shock inputs result in different response spectra

Typical Response Spectra







General Response Spectra (1)

$$x(t)\Big|_{\max} = \frac{1}{m\omega_n}\int_0^t F(\tau)\sin\omega_n(t-\tau)\,d\tau\Big|_{\max}$$

$$S_d = \frac{S_v}{\omega_n}, \qquad S_a = \omega_n S_v$$

$$\dot{z}(t) = -\frac{1}{\omega_d}\int_0^t \ddot{y}(\tau)\,e^{-\zeta\omega_n(t-\tau)}\,[-\zeta\omega_n \sin\omega_d(t-\tau) + \omega_d\cos\omega_d(t-\tau)]\,d\tau$$

$$\dot{z}(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sqrt{P^2+Q^2}\,\sin(\omega_d t - \phi)$$

General Response Spectra (2)

$$P = \int_0^t \ddot{y}(\tau)\, e^{\zeta\omega_n t} \cos \omega_d \tau \, d\tau$$

$$Q = \int_0^t \ddot{y}(\tau)\, e^{\zeta\omega_n t} \sin \omega_d \tau \, d\tau$$

$$\phi = \tan^{-1}\left\{\frac{-(P\sqrt{1-\zeta^2} + Q\zeta)}{(P\zeta - Q\sqrt{1-\zeta^2})}\right\}$$

$$S_v = |\dot{z}(t)|_{\max} = \left|\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sqrt{P^2+Q^2}\right|_{\max}$$

$$S_d = |z|_{\max} = \frac{S_v}{\omega_n}; \qquad S_v = |\dot{z}|_{\max}; \qquad S_a = |\ddot{z}|_{\max} = \omega_n S_v$$

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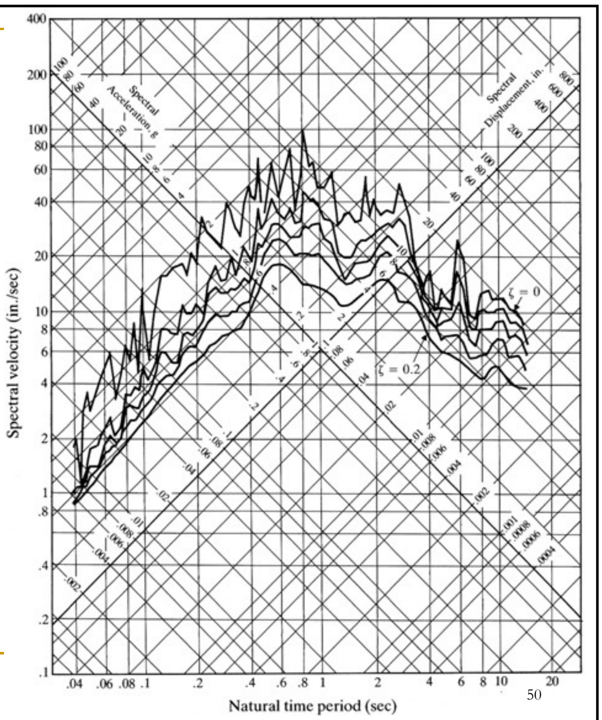
Typical Response Spectrum Subjected to Earthquake



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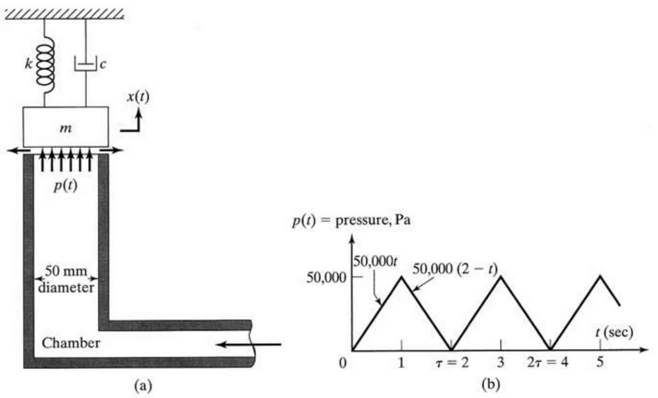
Part VI: Simple Problems

Problem 1. Periodic Vibration of a Hydraulic Valve (Rao 4.1)



In the study of vibrations of valves used in hydraulic control systems, the valve and its elastic stem are modeled as a damped spring-mass system, as shown in Fig. 4.1(a). In addition to the spring force and damping force, there is a fluid pressure force on the valve that changes with the amount of opening or closing of the valve. Find the steady-state response of the valve when the pressure in the chamber varies as indicated in Fig. 4.1(b). Assume $k = 2500$ N/m, $c = 10$ N-s/m, and $m = 0.25$ kg.

Problem 2. Response of a Structure under Impact (Rao 4.4, 4.5)

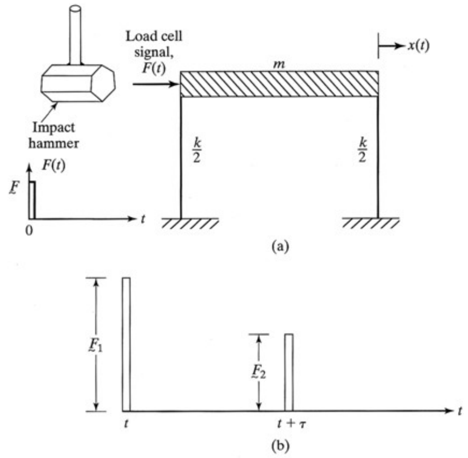
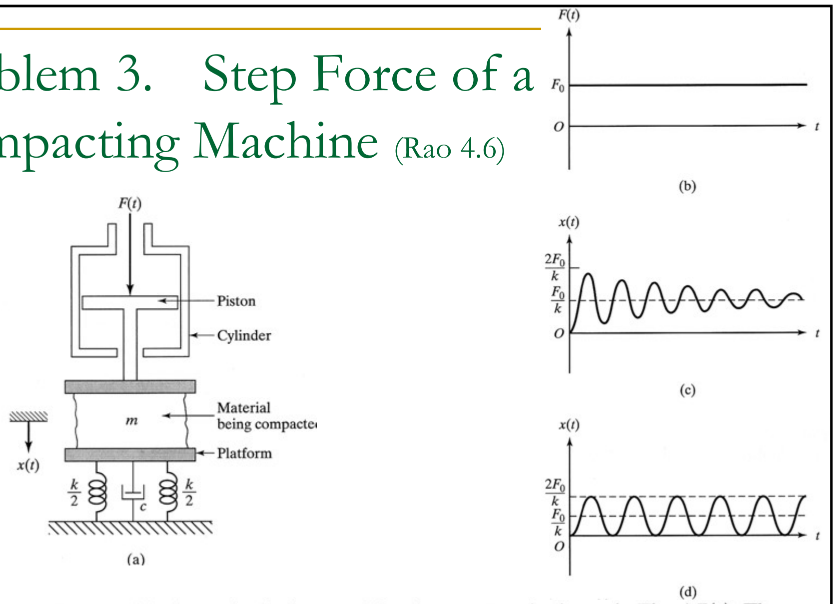


FIGURE 4.5 Structural testing using an impact hammer.

In the vibration testing of a structure, an impact hammer with a load cell to measure the impact force is used to cause excitation, as shown in Fig. 4.5(a). Assuming $m = 5$ kg, $k = 2000$ N/m, $c = 10$ N-s/m and $\underset{\sim}{F} = 20$ N-s, find the response of the system.

Problem 3. Step Force of a Compacting Machine (Rao 4.6)

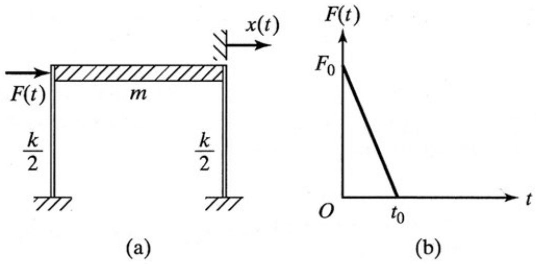


A compacting machine, modeled as a single degree of freedom system, is shown in Fig. 4.7(a). The force acting on the mass m (m includes the masses of the piston, the platform, and the material being compacted) due to a sudden application of the pressure can be idealized as a step force, as shown in Fig. 4.7(b). Determine the response of the system.

Problem 4. Blast Load on a Building Frame (Rao 4.10)

$$F(\tau) = F_0\left(1 - \frac{\tau}{t_0}\right) \text{ for } 0 \le \tau \le t_0$$

$$F(\tau) = 0 \qquad \text{for } \tau > t_0$$



A building frame is modeled as an undamped single degree of freedom system (Fig. 4.11a). Find the response of the frame if it is subjected to a blast loading represented by the triangular pulse shown in Fig. 4.11(b).

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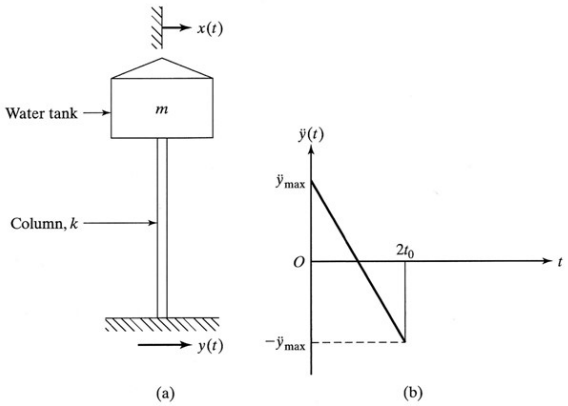
Problem 5: Water Tank Subjected to Base Acceleration (Rao 4.12)



The water tank, shown in Fig. 4.13(a), is subjected to a linearly varying ground acceleration as shown in Fig. 4.13(b) due to an earthquake. The mass of the tank is m, the stiffness of the column is k, and damping is negligible. Find the response spectrum for the relative displacement, $z = x - y$, of the water tank.

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Problem 6. Response of a Building Frame to an Earthquake (Rao. 4.13)

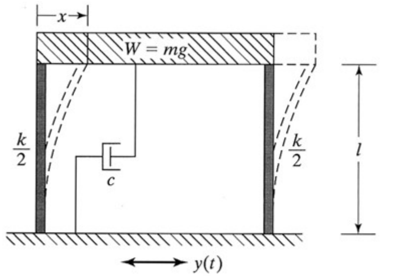

FIGURE 4.17 Building frame subjected to base motion.

A building frame has a mass of 6,800 kg and two columns of total stiffness *k*, as indicated in Fig. 4.17. It has a damping ratio of 0.05 and a natural time period of 1.0 sec. For the earthquake characterized in Fig. 4.15, determine the following:

Part VII: Youtube Demonstrations